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Coherent and BCS-Type Quantum States of Dark Polaritons

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Abstract—We discuss a cavity-based model for coherent quantum states of dark polaritons, i.e., hybrid excitations of matter and light in the presence of electromagnetically induced transparency. Moreover, BCS-type collective atomic quantum states are introduced, which provide a prototype for a condensed state of the coupled system of three-level atoms and light.

1. PROLOGUE

It is our pleasure to contribute to this volume honoring Herbert Walther on the occasion of his 70th birthday. As a diploma student in his laboratory, one of the authors (M.W.) was introduced to the fascinating field of quantum optics.

2. INTRODUCTION

In a two-level system, hybrid atom–light states have long been known as "dressed states" [1]. While their lifetime is limited due to spontaneous decay of the upper electronic state, the concept of dark polaritons as novel, long-lived atom–light states for an ensemble of three-level atoms coupled to a bichromatic laser field was recently introduced by Fleischauer and Lukin [2]. At the heart of these coupled excitations are the well-known dark states [3]. Under two-photon resonance, dense atomic media due to quantum interference can become transparent to light, a phenomenon also known as electromagnetically induced transparency (EIT) [4]. Dark-state media exhibit very interesting properties, such as extremely slow group velocities [5]. Dark state polaritons are quasiparticles that are associated with slow light propagation. It has recently been suggested that such media can be used to store and manipulate quantum information in atomic ensembles [6]. While in the context of quantum information one is interested in fixed excitation numbers, we are here discussing quantum states where the excitation number is subject to quantum fluctuations.

Atomic two-level ensembles coupled to an optical field were first studied in the context of superradiance by Dicke [7]. The thermal equilibrium of the Dicke model was investigated by Hepp and Lieb [8]. Using a constant excitation number and a BCS-type two-level ansatz, Eastham and Littlewood predicted a Bose–Einstein condensate of cavity polaritons for an exciton system in thermal equilibrium [9].

In this work, we discuss coherent states of dark polaritons for an ensemble of three-level atoms coupled to two optical fields. We restrict ourselves to nonpropagating optical fields; i.e, we use a cavity-based model. The obtained hybrid atom–light quantum state can be written as the product of a dark-atomic coherent wavefunction and an optical coherent state. In analogy to optics, it is anticipated that these quantum states are comparatively classical states of the combined system of atoms and light. In the limit of a large photon number, we find that, for the same optical field, the atomic ensemble is also dark for a BCS-type state, which suggests that both atomic states are very similar. Special interest in these BCS-type states is due to the fact that they represent a product of many one-particle states; i.e., they are a macroscopic atomic quantum state. An interesting question for future work is whether one can observe a phase transition of a thermal state to such a well-ordered coherent state. It is anticipated that, in thermal equilibrium, such a phase transition is possible between a thermal state and a Bose–Einstein condensate of dark polaritons.

In the following section, we give a specific model for an ensemble of three-level atoms coupled to one classical and one quantum field. Section 4 then considers the known case of "Fock" polariton states with fixed excitation numbers. Section 5 then discusses coherent and BCS-type quantum states, both of which have variable numbers of dark polaritons. We finally conclude in Section 6.

3. STATEMENT OF THE PROBLEM

We consider an ensemble of N identical three-level atoms, as shown in Fig. 1. The atoms have two long-lived ground states $|g_1\rangle$ and $|g_2\rangle$, e.g., two Zeeman or hyperfine states of an electronic ground state and one spontaneously decaying electronically excited state $|e\rangle$. The atoms are subject to two optical fields. For the sake of simplicity, we shall not consider propagation effects but rather the situation of single-mode stationary optical fields, e.g., as realized in an optical ring cavity [10]. The atomic level $|g_1\rangle$ is coupled to $|e\rangle$ via a fully quantized (weak) signal field, where G denotes the corresponding (one-photon) coupling constant and $a^\dagger$ and a the photon creation and annihilation operators. Furthermore, the atomic levels $|g_2\rangle$ and $|e\rangle$ are coupled by a

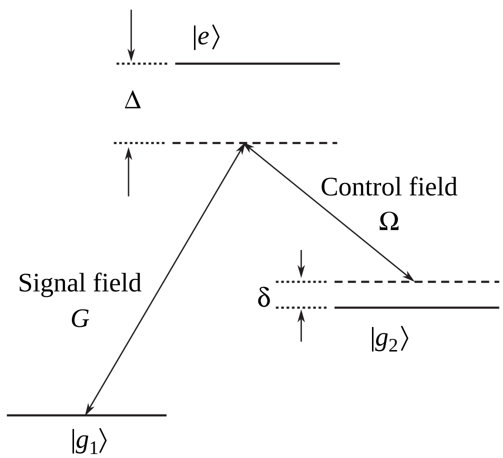

Fig. 1. Three-level system coupled to a (quantum) signal field and a (classical) control field.

stronger, classical control field with Rabi frequency Ω. These couplings are described by the off-diagonal elements of the following interaction-picture Hamiltonian:

$$H_{\text{eff}} = (\Delta - i\Gamma/2)|e\rangle\langle e| - \delta|g_2\rangle\langle g_2| \\ + Ga|e\rangle\langle g_1| + Ga^\dagger|g_1\rangle\langle e| + \Omega|e\rangle\langle g_2| + \Omega^*|g_2\rangle\langle e|, \quad (1)$$

where Δ and δ denote the detunings from the one- and two-photon resonance conditions, respectively. Note that for this definition of the Rabi frequency $I/I_S = 8\Omega^2/\Gamma^2$. Furthermore, the relaxation of the excited state $|e\rangle$ has been accounted for by introducing a non-Hermitian term: $-i\Gamma/2$. For the case of a single atom, the Hamiltonian can be straightforwardly be diagonalized. For $\delta = 0$, one then obtains one dark state with eigenvalue zero, which is a coherent superposition of the two ground states that does not couple to the upper state due to destructive quantum interference, and also two eigenstates coupling to the optical fields. The situation is more involved for the case of an atomic ensemble, which we will consider in the next sections.

4. DARK POLARITONS WITH FIXED QUASIPARTICLE NUMBERS

In the context of the search for the quantum memories of photons, Fleischauer and Lukin gave solutions for the above Hamiltonian for states with fixed quasiparticle number (and also for the more general case of propagating fields, which we shall not consider here) [2]. Our treatment in this section is closely related to their derivation. For the following considerations, both the number of photons in the signal mode and the number of atoms in state $|g_2\rangle$ are of special relevance. If we assume that initially all atoms are prepared in state $|g_1\rangle$, the state $|g_2\rangle$ can only be populated by the absorption of a photon of the signal mode and interaction with the classical control field. Generally speaking, during absorption and stimulated emission processes in a closed three-level system (with no spontaneous decay occurring) the number of quasiparticles N_Q, which we shall define as the sum of the number of signal photons and the number of atoms in state $|g_2\rangle$, remains constant.

Let us define the "vacuum state" as the state with all atoms in $|g_1\rangle$ and no photons in the signal mode, a state for which the quasiparticle number clearly equals zero. A state with a single quasiparticle can be generated in principle, e.g., by generating one signal photon or by decreasing the number of atoms in the state $|g_1\rangle$ by one and simultaneously increasing the number of atoms in the state $|g_2\rangle$ by one. For a suitable linear combination of those two possible excitations, the ensemble remains in a dark superposition state. More generally, one can show that, for $\delta = \Delta = 0$, the collective dark state with N_Q quasiparticles is obtained when applying a suitable quasiparticle operator N_Q times to the vacuum state

$$|\Psi(N_Q)\rangle = \frac{1}{\sqrt{N_Q!}}(\hat{Q}_{DS}^\dagger)^{N_Q} \underbrace{|g_1\rangle_1|g_1\rangle_2\ldots|g_1\rangle_N|0\rangle}_{\text{vacuum}}, \quad (2)$$

where the quasiparticle operator is given by

$$\hat{Q}_{DS}^\dagger = \cos\theta(\Omega)\left(\frac{a^\dagger}{\sqrt{\hat{n}+1}}\right) - \sin\theta(\Omega)\hat{S}_{21}. \quad (3)$$

In the latter formula,

$$\tan\theta(\Omega) = G\sqrt{N}/\Omega \quad (4)$$

and

$$\hat{S}_{21} = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}|g_2\rangle_{jj}\langle g_1|. \quad (5)$$

One can verify that all number states created by $\hat{Q}_{DS}^\dagger$ are dark states that do not couple to the excited state. Moreover, they are eigenstates of the interaction Hamilton with eigenvalues of zero. In the considered limit of many less photons than atoms, the operators $\hat{Q}_{DS}$ and $\hat{Q}_{DS}^\dagger$ obey bosonic commutation relations, and we can associate with it bosonic quasiparticles, which are termed (Fock-state) dark polaritons.

The following explicit form for these Fock-type dark polariton states can be derived [2]:

$$|\Psi_D(N_Q)\rangle \\ = \sum_{k=0}^{N_Q}\sqrt{\binom{N_Q}{k}}(-\sin\theta)^k(\cos\theta)^{N_Q-k}|g_2^k\rangle|N_Q-k\rangle, \quad (6)$$

where $|g_2^k\rangle$ denotes the atomic state given by applying the spin-flip operator k times to the vacuum state and where $|N_Q - k\rangle$ is a photonic Fock state.

5. COHERENT AND BCS-TYPE DARK POLARITON STATES

The polariton Fock states discussed in the previous sections have a constant number of dark state polaritons. Therefore, the absolute phase is totally random. In order to obtain a state with a well-defined phase, we form a suitable superposition of states with different numbers of dark-state polaritons. In analogy to coherent photon states, we define a coherent state of dark polaritons by

$$|\Psi_{D,\gamma}\rangle = \sum_n c_n |\Psi_D(n)\rangle, \qquad (7)$$

with the weights $c_n = \frac{\gamma^n}{\sqrt{n!}}\exp\left[\frac{-|\gamma|^2}{2}\right]$, which is a poission distribution with $|\gamma|^2 = N_{Q,\,\text{av}}$ as the mean number of quasiparticles. One now obtains

$$|\Psi_{D,\gamma}\rangle = e^{-\frac{|\gamma|^2}{2}} \sum_{n,k} \frac{\gamma^n(-\sin\theta)^k(\cos\theta)^{n-k}}{\sqrt{k!(n-k)!}} |n-k\rangle|g_2^k\rangle. \quad (8)$$

With $m = n - k$, we can separate this formula into two independent terms:

$$|\Psi_{D,\gamma}\rangle = e^{-\frac{|\gamma|^2}{2}} \sum_k \frac{(-\gamma\sin\theta)^k}{\sqrt{k!}} |g_2^k\rangle \sum_m \frac{(\gamma\cos\theta)^m}{\sqrt{m!}} |m\rangle. \quad (9)$$

These states are obviously the independent product of a (dark) coherent atomic state and a coherent state of the light mode. We are usually interested in states with relatively large number of particles, for which the fluctuations in the polariton number are small. We can then denote the coherent state of a dark polariton as

$$\begin{aligned} &|\Psi_{D,\sqrt{N_{Q,\,\text{av}}}}\rangle \\ &= |\Psi_{\text{Atom}}(-\sqrt{N_{Q,\,\text{av}}}\sin\theta)\rangle|\Psi_{\text{Light}}(\sqrt{N_{Q,\,\text{av}}}\cos\theta)\rangle, \end{aligned} \qquad (10)$$

or, if we denote the coherent photon state as $|\alpha\rangle$ ($\equiv |\sqrt{N_{Q,\,\text{av}}}\cos\theta\rangle$), we arrive at

$$|\Psi_{D,\sqrt{N_{Q,\,\text{av}}}}\rangle = |\Psi_{\text{Atom}}(-\sqrt{N_{Q,\,\text{av}}}\sin\theta)\rangle|\alpha\rangle. \qquad (11)$$

In contrast to dark polariton states with fixed polariton numbers, here, both atomic and photonic contributions have a fixed phase. The photonic part of the above quantum state is relatively stable against decoherence. The atomic part still involves atomic correlations and certainly is not of a straightforward explicit form. In case of the existence of atomic states that do not involve atom correlations, which are also dark for the same light field, it seems much more physically reasonable that the hybrid atom–light system will evolve into such a state in real experimental situations. With a coherent state for the quantum state of the signal field, using $a|\alpha\rangle = \alpha|\alpha\rangle$ we can explicitly write down the Hamiltonian in the atomic subspace. While the above treatment is rigorously valid only for $\delta = \Delta = 0$, we shall more generally also allow for nonzero detunings. We arrive at a Hamiltonian in the atomic basis $H = \sum_{j=1}^N H_j$, where

$$H_j = \begin{pmatrix} 0 & G\alpha^* & 0 \\ G\alpha & \Delta - i\frac{\Gamma}{2} & \Omega \\ 0 & \Omega^* & -\delta \end{pmatrix}_j. \qquad (12)$$

This simple form of the Hamiltonian suggests that a total wavefunction can be found with the ansatz

$$\begin{aligned} |\Psi_C\rangle &= \prod_{j=1}^N (c_1|g_1\rangle_j + c_e|e\rangle_j + c_2|g_2\rangle_j)|\alpha\rangle \\ &\equiv \prod_{j=1}^N |\Psi_j\rangle|\alpha\rangle, \end{aligned} \qquad (13)$$

which is a product of a BCS-type atomic state and a coherent photon state. We note that a similar type of ansatz was used in earlier work on two-level systems, for which Bose–Einstein condensation was found theoretically [9]. The number of quasiparticles is fixed by the condition $N_{Q,\,\text{av}} = |\alpha|^2 + N|c_2|^2$, which yields $\alpha = \sqrt{N_{Q,\,\text{av}} - N|c_2|^2}e^{i\varphi}$. Without loss of generality, we shall set the phase $\varphi = 0$. To obtain the coefficients c_i, we solve the eigenvalue equation

$$\begin{aligned} 0 &= \sum_{j=1}^N (H_j - \lambda)((|\Psi_1\rangle|\Psi_2\rangle\ldots|\Psi_N\rangle)|\alpha\rangle \\ &= \sum_{j=1}^N (H_j - \lambda)|\Psi_j\rangle\left(\prod_{i \neq j}|\Psi_i\rangle\right)|\alpha\rangle. \end{aligned} \qquad (14)$$

Since every single factor has to vanish, the above many-body problem can be reduced to a single-body problem that is valid for every atom j:

$$\begin{aligned} 0 &= (H_j - \lambda)|\Psi_j\rangle \\ &= \begin{pmatrix} -\lambda & G\alpha & 0 \\ G\alpha & \Delta - \lambda - i\frac{\Gamma}{2} & \Omega \\ 0 & \Omega & -\delta - \lambda \end{pmatrix}_j \begin{pmatrix} c_1 \\ c_e \\ c_2 \end{pmatrix}_j. \end{aligned} \qquad (15)$$

One of the three eigenvalues has a value near zero for small δ, and is to first order given by

$$\lambda = \frac{-\delta|\alpha|^2 G^2}{\Omega^2 + G^2|\alpha|^2}. \qquad (16)$$

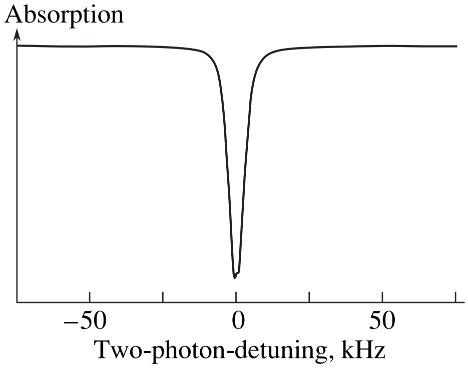

Fig. 2. Experimentally recorded dark resonance in a rubidium buffer (^{87}Rb) gas cell as a function of two-photon detuning. The observed resonance width is 5.4 kHz.

Note that the atoms are not exactly dark for the light field for $\delta \neq 0$. The coefficients for this dark (or, more generally, "gray") eigenstate are

$$\begin{pmatrix} c_1 \\ c_e \\ c_2 \end{pmatrix} = \frac{1}{\sqrt{G^2|\alpha|^2 + \Omega^2}} \begin{pmatrix} -\Omega \\ \dfrac{\delta\alpha\Omega}{G^2|\alpha|^2 + \Omega^2} \\ G\alpha \end{pmatrix}. \tag{17}$$

If $\delta = 0$, the eigenvalue λ reaches exactly zero and the photonic and atomic degrees remain uncoupled, which is easily verified. We can eliminate the signal field photon number using the relation $N_{Q,\,\mathrm{av}} = |\alpha|^2 + N|c_2|^2$, and in the limits $NG^2 \gg \Omega$, $\Omega \gg G|\alpha|$ arrive at the simple form

$$c_1 = -\sqrt{1 - \frac{N_{Q,\,\mathrm{av}}}{N}},$$
$$c_e = \frac{\delta}{\Omega}\sqrt{\frac{N_{Q,\,\mathrm{av}}}{N}\left(1 - \frac{N_{Q,\,\mathrm{av}}}{N}\right)}, \tag{18}$$
$$c_2 = \sqrt{\frac{N_{Q,\,\mathrm{av}}}{N}}.$$

Let us here also give the energy per quasiparticle, which equals

$$\mu = \lambda\frac{N}{N_{Q,\,\mathrm{av}}} = -\frac{\delta N}{N + |\alpha|^2 + \Omega^2/G^2}. \tag{19}$$

6. CONCLUSIONS

For an ensemble of three-level atoms in a high-finesse optical cavity, we have constructed coherent states of dark polaritons, which have a defined phase for both the atomic and photonic degrees of freedom. This is in contrast to the well-known Fock-type polariton states, where only the relative phase of the contributions is determined. However, the coherent polariton states are subject to quantum fluctuations in the number of polariton excitations. We have, moreover, introduced dark polaritons with a BCS-type atomic contribution (and a coherent photonic contribution). These states constitute a possible collective state for a condensate of dark polaritons. In the future, it would be interesting to study thermal excitations, and even more interesting to experimentally observe a Bose–Einstein phase transition between a thermal state and a dark polarition condensate. The experimentally challenging part here is to establish a true thermal equilibrium between the photonic and atomic degrees of freedom. Efforts towards the realization of such a type of phase transition are underway in our Tübingen laboratory.

An interesting theoretical question is whether one can generalize coherent and BCS-type dark excitations to traveling waves. Indeed, it seems very likely that those types of states are the best approximations to the actual quantum states present in usual buffer-gas slow-light experiments. As an example, for a typical dark resonance, Fig. 2 shows an experimentally observed spectrum recorded in our laboratory with a thermal buffer-gas cell. The optical transmission through the cell lessens when the laser difference frequency is detuned away from two-photon resonance. Similarly, any fluctuations or oscillations in the optical frequency that lead to a spectral broadening beyond the dark resonance width also lower the cell transmission. An interesting question is whether such an experiment is related to the well-known phenomenon of the ac conductivity of superconductors being lower than the (infinite) dc conductivity.

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