Optical multiphoton lattices

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We propose a scheme that allows us to Fourier-synthesize arbitrarily shaped periodic potentials for atoms. The method is based on the dispersion connected with higher order multiphoton Raman processes, where a suitable combination of laser frequencies eliminates unwanted standing wave effects. Future application of the scheme could range from blazed atomic beam splitters up to atomic quantum ratchets.

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Cold atoms can be trapped in the periodic potential induced by the ac Stark shift in optical standing waves [1]. These systems, the so called optical lattices, have found broad applications ranging from the observation of Bloch oscillations up to quantum computing. In conventional optical lattices, the trapping potential is sinusoidal for a given atomic state, while the spatial periodicity for counterpropagating beams equals half the optical wavelength. Some authors studied superlattices based on the spatial beating of two neighboring trapping frequencies [2]. A certain class of asymmetric, dissipative potentials can be realized in a grey optical lattice configuration [3].

An interesting question is whether one can realize a more general class of periodic potentials for cold atoms. Of special interest would be, e.g., conservative asymmetric periodic potentials, which can allow for studies of quantum ratchets with Bose-condensed atoms. Asymmetric potentials have been studied in detail in the classical limit both in biological systems within the context of molecular motors and also in solid state systems [4]. It has been noted recently that the quantum regime here exibits several additional intriguing features [5]. It is clear that any periodic potential can be Fourier-decomposed into an infinite sum of spatial harmonics of a sinusoidal potential. However, it seems impractical to use harmonics of a laser frequency to Fourier-synthesize a potential when a larger number of Fourier components is required. A different approach would be to use beams of the same wavelength inclined under different angles, although such a setup would be difficult to accurately align and phase stabilize for a larger number of beams.

We here propose to trap atoms in a periodic potential induced by higher order multiphoton Raman processes. Our scheme benefits from the fact that an $2N$th order multiphoton process can yield a spatial periodicity of $\lambda/2N$, where λ denotes the laser wavelength. A suitable combination of optical frequencies allows to avoid unwanted standing wave effects. When combining the potentials induced by multiphoton processes of different order, arbitrarily shaped periodic potentials can be Fourier-synthesized.

Multiphoton spectroscopy has a long history both in the optical and radiofrequency regimes [6,7]. In a recent experiment, multiphoton Raman transfer with up to 50 photons was observed between two ground state Zeeman resonances driven by milliwatts of laser power [8]. While all those experiments studied multiphoton absorption, the present proposal is based on dispersive properties of suitable higher order transitions [9].

In a conventional optical lattice, the trapping potential is given by $V=-(\alpha/2)|E(\mathbf{r})|^2$, where α denotes the dynamic atomic polarizability, so that, e.g., in a standing wave oriented along the z-axis $V\propto I(z)=I_0\cos^2 kz=I_0(1+\cos 2kz)/2$, yielding the well known spatial periodicity of $\lambda/2$. Let us now address the question of whether we can obtain a higher spatial periodicity using multiphoton transitions. The perhaps simplest approach would be to use a Doppler-sensitive Raman transition between two stable ground states $|g_0\rangle$ and $|g_1\rangle$ with the electronically excited state $|a\rangle$ acting as an intermediate level, as shown in Fig. 1(a) [10]. The transition is driven by two counterpropagating optical beams of frequencies ω and ω', where $\omega'\simeq\omega=c\cdot k$. When the one-photon detuning Δ is large, one can adiabatically eliminate the upper state $|a\rangle$ and obtain an effective one-photon transition between ground states $|g_0\rangle$ and $|g_1\rangle$ with effective frequency $\omega_{\mathbf{eff}}=\omega-\omega'$ and wavevector $\mathbf{k_{eff}}=\mathbf{k}-\mathbf{k}'\simeq 2\mathbf{k}$. We are interested in dispersive properties of this coupling. As we are aiming at a lattice potential, consider the atomic potential obtained when retroreflecting both optical beams. It is easy to show that when a nonzero two-photon δ detuning is used, e.g., atoms in state $|g_0\rangle$ experience a fourth order energy shift $V\propto\cos^2 k_{\mathbf{eff}}\cdot z=(1+\cos 4kz)/2$. We indeed obtain a term with spatial periodicity of twice that of a conventional lattice. Interestingly, this behavior cannot be explained simply by the nonlinearity of the higher order transition, i.e., V is not proportional to $I(z)I'(z)$. Since the system reduces to an effec-

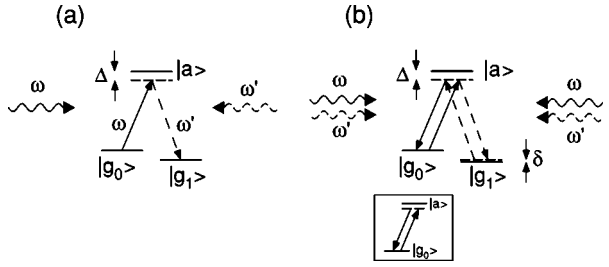

FIG. 1. (a) Level scheme for Doppler-sensitive stimulated Raman transfer between two ground states $|g_0\rangle$ and $|g_1\rangle$ through an excited state $|a\rangle$. (b) Virtual processes contributing to dispersive properties of a Doppler-sensitive Raman transition. Atoms in state $|g_0\rangle$ experience a light-induced potential with spatial periodicity $\lambda/4$ caused by fourth-order processes. However, the usual standing wave potential of periodicity $\lambda/2$ induced by second order processes, as indicated in the inset, here dominates.

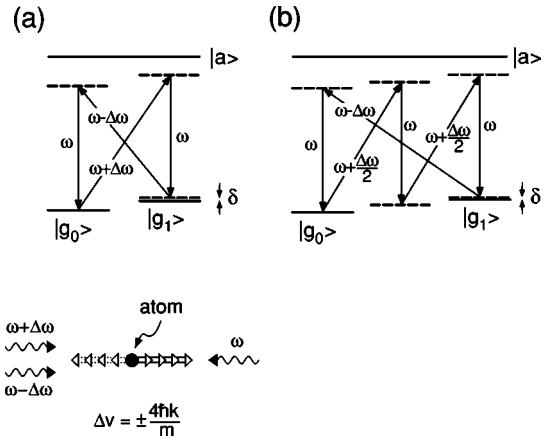

FIG. 2. (a) Proposed scheme for generation of an optical multiphoton lattice. For a fourth-order periodic potential, atoms are driven by beams of frequencies $\omega+\Delta\omega$ and $\omega-\Delta\omega$, and a counterpropagating beam of frequency ω. In each fourth-order process, the velocity of the atom changes by $\Delta v=\pm 4\hbar k/m$. (b) Generalization to sixth's order processes (for which $\Delta v=\pm 6\hbar k/m$), yielding a lattice potential of periodicity $\lambda/6$.

tive two-level system, it also follows that atoms in state $|g_1\rangle$ experience an energy shift of equal magnitude, but with opposite direction. The drawback of the scheme of Fig. 1(b) clearly is that here the periodic potential induced by second order processes as shown in the inset of the figure (i.e., the normal ac Stark shift from standing waves) will dominate over fourth order processes. Thus the desired effect of a higher periodicity will in this simple scheme be masked by a conventional lattice potential.

In the proposed scheme, standing wave effects are eliminated by the use of a degenerate double Raman scheme, and this is possible with a minimum of three laser frequencies. We will begin by discussing a fourth-order process, while the generalization to arbitrarily high orders is discussed below. In the scheme shown in Fig. 2(a), a three-level atom is irradiated with beams of optical frequencies $\omega+\Delta\omega$ and $\omega-\Delta\omega$ from the left and a beam with frequency ω from the right. It is clear that standing wave effects are now absent. Remarkably, atoms in, e.g., ground state $|g_0\rangle$ nevertheless do experience the desired fourth-order energy shift with spatial periodicity of twice that of a conventional optical lattice. Intuitively, this can be seen when considering the momentum transfer acquired by an atom during such a closed loop four-photon process. An atom initially in state $|g_0\rangle$ can undergo virtual processes (as indicated in the figure) in which (i) a photon from the beam with frequency $\omega+\Delta\omega$ is absorbed and (ii) emitting a photon into the mode with frequency ω. At this point, the atom has acquired the momentum of two photon recoils and is in a virtual state with energy close to that of state $|g_1\rangle$. A stable final state is reached when in addition (iii) absorbing a photon of frequency $\omega-\Delta\omega$ and (iv) emitting a further photon into the mode with frequency ω, which takes the atom back into ground state $|g_0\rangle$. After such a four-photon process, the internal state is unchanged, and the atom has aquired a momentum of $4\hbar k$ ($-4\hbar k$ for a reverse process). While in a standing wave virtual processes with momentum transfer of $2\hbar k$ and $-2\hbar k$ respectively yield $a\propto(1+\cos 2kz)$ dependence of the lattice potential in the spatial domain, we here expect a (state-dependent) potential $\propto(1+\cos 4kz)$, which has double spatial periodicity. A selection of the desired multiphoton process is achieved by choice of appropiate frequencies for the optical beams. It is assumed that the frequency difference $\Delta\omega$ is close to the energy splitting between ground states $|g_0\rangle$ and $|g_1\rangle$, so that the two-photon detuning δ is much smaller than $\Delta\omega$. This suppresses same order processes of other type than the one shown shown in Fig. 2(a) (as, e.g., the process obtained when exchanging the roles of $\omega-\Delta\omega$ and $\omega+\Delta\omega$). We moreover assume that $\Delta\omega$ is large compared to the lattice vibrational frequencies.

Let us next discuss a generalization of the multiphoton lattice to higher orders. A lattice with spatial period $\lambda/2N$ (with N integer) requires a $2N$th order multiphoton transition, as can be realized by replacing the beam of frequency $\omega+\Delta\omega$ of the four-photon scheme of Fig. 2(a) by a beam with frequency $\omega+\Delta\omega/(N-1)$ for $N\geqslant 2$. In this way, energy conservation is achieved only for the desired order transition. Similar as previously it is assumed that $\delta\ll\Delta\omega/(N-1)$, in which case we expect that the process in which one of the intermediate detunings is δ [as shown in Fig. 2(b) for $N=3$] dominates over other multiphoton process of similar order. We note that to achieve reasonable potential depths it is now expected to be of importance that the energy splitting between ground states $|g_0\rangle$ and $|g_1\rangle$ is relatively small, i.e., comparable to the two-photon Rabi frequency. This can be realized experimentally by using different Zeeman sublevels of a hyperfine ground state for states $|g_0\rangle$ and $|g_1\rangle$. When the laser detuning from the upper electronic state is much larger than the upper state hyperfine splitting, transitions are only possible between neighboring ground state Zeeman levels [11]. We are aware that also in this situation more than single upper state can be relevant. Note that in a recent experiment studying multiphoton Raman transitions a transfer between Zeeman ground states with up to 50 exchanged photons has been observed [8]. The present scheme involves multiphoton dispersion, which requires (a) the presence of a nonzero Raman-detuning δ and (b) an additional two-photon exchange tuned degenerately to the $2(N-1)$ multiphoton ladder [see Fig. 2(b)]. As in the four-photon scheme, standing wave effects can again be avoided.

We have in a semiclassical picture calculated the light shift potential for the multiphoton lattice. Assume a Hamiltonian $H=\hbar\omega_a|a\rangle\langle a|+\hbar\omega_z|g_1\rangle\langle g_1|-e\mathbf{r}\cdot\mathbf{E}$, where ω_a and ω_z denote the energy of states $|a\rangle$ and $|g_1\rangle$ respectively in frequency units when measuring their energies relatively to $|g_0\rangle$. The laser electric field is $\mathbf{E}=\mathbf{E}_{0,0}\cos(\omega t+kz)+\mathbf{E}_{0,+}\cos[(\omega+\Delta\omega/(N-1))t-kz]+\mathbf{E}_{0,-}\cos[(\omega-\Delta\omega)t-kz]$. We assume that $\Delta=\omega_a-\omega\gg\Delta\omega/(N-1)\gg\delta=\Delta\omega-\omega_z$. To solve the time-dependent Schrödinger equation, we use an interaction picture corotating with the optical field. To straightforwardly account for all virtual levels of the multiphoton problem, we introduce separate coefficients for probability amplitudes of a given internal level rotating with different frequencies. We use an ansatz

$$|\psi\rangle = \sum_{n=0}^{N-1} e^{-i[n\Delta\omega/(N-1)]t}(g_{n,0}|g_0\rangle + g_{n,1}|g_1\rangle + a_n e^{-i\omega t}|a\rangle), \tag{1}$$

where $g_{0,1}=g_{N-1,0}=0$. The amplitudes g_{n,s_n} and a_n are only slowly varying with time. We assume a coupling to only occur between terms rotating with the same frequency, which is valid for slowly varying probability amplitudes. This assumption can be considered as a generalization of the usual rotating wave approximation, and yields a coupling of probability amplitudes corresponding to the schematic representation of Fig. 2(b). Since Δ was assumed to be large (more precisely, $\dot{a}_n \ll \Delta \cdot a_n$), we can directly adiabatically eliminate all upper state amplitudes. This yields the set of equations

$$\dot{g}_{0,0} = \frac{i}{4\Delta}\left[(|\Omega_{0,0}|^2 + |\Omega_{0,+}|^2)g_{0,0} + \Omega_{0,0}\Omega_{-,1}^* e^{2ikz} g_{N-1,1} + \sum_{s_1=0}^{1} \Omega_{+,0}\Omega_{0,s_1}^* e^{-2ikz} g_{1,s_1}\right], \tag{2a}$$

while, for $1 \leqslant n \leqslant N-2$,

$$\dot{g}_{n,s_n} = i\left(\frac{n\Delta\omega}{N-1} - s_n\omega_z\right)g_{n,s_n} + \frac{i}{4\Delta}\left[\sum_{s_n'=0}^{1}(\Omega_{0,s_n}\Omega_{0,s_n'}^* + \Omega_{+,s_n}\Omega_{+,s_n'}^*)g_{n,s_n'} + \sum_{s_{n-1}'=0}^{1}\Omega_{0,s_n}\Omega_{+,s_{n-1}'}^* e^{2ikz} g_{n-1,s_{n-1}'} + \sum_{s_{n+1}'=0}^{1}\Omega_{+,s_n}\Omega_{0,s_{n+1}'}^* e^{-2ikz} g_{n+1,s_{n+1}'}\right], \tag{2b}$$

and finally

$$\dot{g}_{N-1,1} = i(\Delta\omega - \omega_z)g_{N-1,1} + \frac{i}{4\Delta}\left[(|\Omega_{0,1}|^2 + |\Omega_{-,1}|^2)g_{N-1,1} + \sum_{s_{N-2}=0}^{1}\Omega_{0,1}\Omega_{+,s_{N-2}}^* e^{2ikz} g_{N-2,s_{N-2}} + \Omega_{-,1}\Omega_{0,0}^* e^{-2ikz} g_{0,0}\right]. \tag{2c}$$

In these formulas, one directly identifies terms yielding a position independent ac-Stark shift. For example, from the probability amplitude $g_{0,0}$ to lowest order we obtain the usual (second order) frequency shift

$$\Delta\omega_{\mathrm{AC},2} \simeq -\frac{1}{4\Delta}(|\Omega_{0,0}|^2 + |\Omega_{+,0}|^2). \tag{3}$$

This frequency shift is certainly larger than all higher order effects. However, for a collimated laser beam one only obtains an absolute energy shift, which does not prevent one from trapping atoms in a higher order lattice potential. To derive the spatially dependent light shift potential, we use a new interaction picture in which all spatially constant terms are factored out of the Hamiltonian. The equations of motion

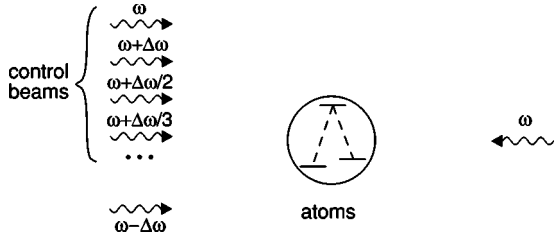

FIG. 3. Scheme for a Fourier-synthesis of arbitrarily shaped periodic potentials.

for the ground state probability amplitudes $\tilde{g}_{n,s_n}$ of this new interaction picture are identical to Eqs. (2a)–(2c) when one omits all terms which do not contain a $\exp(\pm 2ikz)$ factor. We now adiabatically eliminate all intermediate ground state amplitudes $\tilde{g}_{n,s_n}$ with $n \geqslant 1$. By iteration, we can derive a single differential equation for the amplitude $\tilde{g}_{0,0}$ (only position dependent terms are maintained), which can be written as $\tilde{g}_{0,0} = \Delta\omega_{\mathrm{AC},2N}\,\tilde{g}_{0,0}$ with the position-dependent Stark shift

$$\Delta\omega_{\mathrm{AC},2N} = -\frac{1}{(4\Delta)^N}\sum_{s_1,\ldots,s_{N-2}}\left[e^{i2Nkz}\Omega_{0,0}\Omega_{-,1}^* \times \prod_{n=1}^{N-1}\frac{\Omega_{0,s_n}\Omega_{+,s_{n-1}}^*}{s_n\omega_z - \frac{n\Delta\omega}{N-1}} + \text{c.c.}\right], \tag{4}$$

where $s_{N-1}=1$. This expression is our final result for the light shift potential of the multiphoton lattice in the adiabatic limit. The spatial periodicity of the lattice potential is $\lambda/2N$, which is a factor N above that of a conventional lattice. The phase of the periodic potential (i.e., the precise location of, e.g., an antinode) can be adjusted by varying the phase of the optical fields, which results in a phase shift of the corresponding Rabi frequencies in Eq. (4). Since all ground state amplitudes other than $\tilde{g}_{0,0}$ (or $g_{0,0}$) are small (which did allow their adiabatic elimination), the dominating contribution to the position independent part of the Stark shift of $|g_0\rangle$ is given by Eq. (3).

To Fourier-synthesize arbitrarily shaped periodic potentials, it is clear that we have to combine lattice potentials with different spatial periodicities. A possible approach for such a Fourier-synthesis is shown in Fig. 3. Compared to the above discussed single-period scheme [shown e.g., in Fig. 2(b) for $N=3$], the beam with frequency $\omega+\Delta\omega/(N-1)$ is replaced by a series of components ("control beams") with optical frequencies: ω, $\omega+\Delta\omega$, $\omega+\Delta\omega/2$, …, $\omega+\Delta\omega/(N_{\max}-1)$ where the phase and amplitude in each of those components is set to the appropriate value. Together with the additional copropagating beam of frequency $\omega-\Delta\omega$ and one counterpropagating beam of frequency ω a superposition of periodic potentials is formed: The two beams of frequency ω form a conventional lattice of period $\lambda/2$, which corresponds to the fundamental spatial frequency. Further, the copropagating beams of frequency $\omega+\Delta\omega/(N-1)$ (with $N \geqslant 2$) and that with $\omega-\Delta\omega$ together with the counterpropagating beam of frequency ω provide the higher harmonics of periodicity

$\lambda/2N$. The phases and amplitudes of the control beams are adjusted to yield the desired contribution of the corresponding harmonic to the desired overall potential.

Let us next discuss typical experimental parameters. For a laser detuning Δ from the upper state of $\simeq 10^5$ linewidths of the rubidium D2-line and a 0.1 mm beam diameter, we expect a two-photon Rabi frequency $\Omega^2/2\Delta$ near 10 MHz for 100 mW beam power. This detuning is far above the upper state hyperfine splitting, so that Raman coupling only occurs between between neighboring ground state Zeeman levels. The transitions could, e.g., be driven by orthogonal linear optical polarizations with a finite angle of the magnetic bias field to the lasers beam axis, as done in Ref. [8]. The experimental data of that paper showed that for higher order Raman transitions the dependence of the effective Rabi frequency on the Zeeman splitting is quite drastic, which is easily understood in terms of the high nonlinearity of these transitions. One may wish to choose a splitting between Zeeman levels somewhat above or comparable to the two-photon Rabi frequency. We expect a possible potential depth near 100 kHz up to about $N=5$ (yielding a spatial harmonic of period $\lambda/10$). For larger laser intensities, even shorter period lattices should be possible. Though the expected lattice potential depth is considerably smaller than the usual Stark shift induced by the trapping beams (which is of order of the two-photon Rabi frequency), the force induced by the high periodicity lattice exceeds that induced by the usual Stark shift due to intensity gradients along the trapping beam axis by more than two orders of magnitude, if we assume the above noted beam parameters. For the quoted experimental parameters, the expected trap vibrational frequencies are far below the used two-photon detunings $s_n\omega_z - n\Delta\omega/(N-1)$, which ensures that adiabaticity is fulfilled. In a different language, we do not expect resonant Raman processes caused by Doppler shifts associated with the vibrational motion.

The various optical frequencies required for a synthesis of different spatial periods can be derived from a single laser with acoustoptic modulators. Note that when one uses devices with sufficiently high bandwidth, a single modulator per beam direction can be sufficient. The problem of generating different spatial frequencies is then reduced to a synthesis of the appropriate combination of rf drive frequencies of an acoustoptic modulator.

To conclude, we have proposed a method to Fourier-synthesize arbitrarily shaped periodic potentials for atoms. Such periodic potentials have intriguing prospects for the study of quantum ratchets with quantum gases. Other applications could include blazed atomic beam-splitters or novel schemes for quantum computing in optical lattices.

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