Tensorial phases in multiple beam atomic interference

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The atomic tensor polarizability in a pulsed optical field can generate a phase-shift scaling quadratically with the interfering path number in a multiple beam Ramsey experiment. The phase can be interpreted as an internal atomic-state version of the electric (or scalar) Aharonov-Bohm effect. In the absence of classical forces, this nonlinear phase shift causes collapse and revival of the Airy-function-like interference pattern. The technique also holds promise for experiments testing for a permanent electric dipole moment of an atom.

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While in classical mechanics potentials are only convenient mathematical tools, Aharonov and Bohm showed that in quantum mechanics potentials have important physical significance [1]. The essence of the Aharonov-Bohm effect is that phase shifts can be induced in regions with no classical force but a nonzero potential. In their famous proposal, Aharonov and Bohm predicted both a magnetic and an electric effect. The magnetic or vector Aharonov-Bohm effect was experimentally observed with electrons [2]. Neutral particle analogs of the electric or scalar Aharonov-Bohm effect were realized with neutrons [3,4] and atoms [5,6]. In all these experiments the Aharonov-Bohm phase caused a shift of the fringe pattern of two-beam interference experiments. Within the last years the field of interferometry with atoms has undergone rapid progress [7]. Recently, not only two-beam, but also multiple beam atom interferometers have been realized experimentally [8,9], leading to sharply peaked fringe patterns and the observance of collapse and revival induced by a nonlinear phase associated with the photon recoil energy [10,11]. Since then, related effects have been predicted in theoretical studies of the Aharonov-Bohm effect in multiple beam electron interferometers [12].

In this Rapid Communication we report on a study of tensorial phase shifts associated with the internal atomic degrees of freedom in a multiple beam Ramsey interference experiment. Via the tensor contribution to the atomic polarizability, a pulsed off-resonant light field can generate a potential and an accumulated phase shift scaling quadratically with the interfering path number. This nonlinear phase shift is achieved with no classical force acting on the atoms, and represents an example of a generalized electric Aharonov-Bohm phase shift [6,4]. In the absence of the nonlinear phase, the phase difference between adjacent paths is constant for all interfering paths, and the measured interference signal exhibits the sharply peaked Airy-function-like pattern known from common optical multiple beam interference experiments, as, e.g., a Fabry-Perot resonator. When the light intensity is increased to a finite value, the phase difference between adjacent paths is not constant anymore, and the measured fringe pattern collapses as soon as the accumulated quadratic phase between a central and an outermost path becomes significant. However, when the accumulated quadratic phase between neighboring paths reaches an integer multiple of 2\pi, the original fringe pattern is revived. Further interesting effects, such as period doubling, are observed for phase differences of integer fractions of 2\pi. While linear phases simply shift the multiple beam interference pattern, quadratic phase shifts result in collapse and revival effects, thus yielding a clear signature of the scaling of the phases. The quadratic phase causes fringe patterns related to the Talbot images of near field light optics, where nonlinear phase terms appear in the Fresnel approximation of the wave equation [13].

Let us briefly discuss a scheme of the originally proposed electric variant of the Aharonov-Bohm effect as shown in Fig. 1(a). In that gedanken experiment, an electron wave packet is coherently split into two paths, each of which passes through a conducting Faraday cylinder with a field-free region inside. During a time when the wave packets are completely contained in the cylinders, the electrostatic potentials of the cylinders are pulsed onto the values \Phi_1 and \Phi_2, respectively, for a period \tau. In spite of the absence of all forces, the electron wave packets will acquire phase shifts \Delta \varphi_i = -V_i/\tau, with the scalar potentials \Phi_i \equiv -e\Phi_i. While this experiment still remains to be performed with charged particles, the potentials can also be generated by the interaction of a dipole moment with a field [3–6], yielding \( V = -\mathbf{d} \cdot \mathbf{E} \) for an electric dipole moment \( \mathbf{d} \) in an electric field \( \mathbf{E} \). The electric dipole moment of an atom is \( \mathbf{d} = \mathbf{d}_{\text{perm}} + \hat{\alpha} \mathbf{E} \), where \( \hat{\alpha} \) denotes the polarizability tensor. The permanent electric dipole moment \( \mathbf{d}_{\text{perm}} \) of atoms—being so small that it has so far resisted experimental detection—can for now certainly be neglected. A significant atomic electric dipole moment can, however, be induced by the interaction with both static and ac electric fields. The interaction energy then is determined by the static or ac Stark shift. In the case of either a static or also a linearly polarized ac electric field, the original fringe pattern is revived.

![FIG. 1. Schematics of electric Aharonov-Bohm experiments (a) with electrons and two interfering paths and (b) with neutral atoms and five interfering paths.](image-url)
(otherwise, in addition a term linear in \( m_F \) appears) a magnetic sublevel-dependent phase shift of the form

\[
\Delta \varphi_{m_F} = \frac{1}{\hbar} \left( \frac{\alpha_{\text{scalar}}}{2} + \frac{\alpha_{\text{tensor}}}{2} \frac{3m_F^2 - F(F-1)}{F(2F-1)} \right) E^2 \tau, \tag{1}
\]

is accumulated in an interaction time \( \tau \), where \( F \) denotes the atomic total angular momentum [14]. The phase shift is induced with no classical force, provided that the atoms do not experience an electric-field gradient. While the original concept of the electric Aharonov and Bohm effect [1] relied on interfering paths with the same charge passing through regions with different potentials in each of the paths, our experiment is based on a path-dependent atomic polarizability, such that the same electric field can be used for all paths (with different magnetic quantum numbers) and no spatial separations between the paths is required. This represents a generalized form of an electric Aharonov-Bohm phase shift [3–6]. We are aware of the ongoing discussion in the literature about to what extent the resulting phase shift in related spin rotation experiments is a complete analog to the original Aharonov-Bohm concept [4]. As an important extension over previous work, in our experiment the atomic tensor polarizability yields a phase term scaling quadratically with the magnetic quantum number. Compared to the static field case, the tensor polarizability of ground-state alkali-metal atoms can be enhanced by many orders of magnitude by using an optical field with a detuning from resonance smaller or comparable to the upper state hyperfine splitting. This allows the generation of large nonlinear phase shifts already with moderate field strengths. One can read out the tensorial phase very elegantly using an interferometer with the interfering paths in different magnetic sublevels, as shown in Fig. 1(b). Phase shifts due to the scalar polarizability cancel when all interfering paths are in the same electronic state and experience the same electric field.

For a detection of this nonlinear phase, we use the technique of multiple beam Ramsey spectroscopy developed in our laboratory [8]. Atoms from a cesium atomic beam are irradiated by a sequence of two copropagating optical Ramsey beams in a \( \sigma^+ - \sigma^- \) polarization configuration tuned to the \( F = 4 \rightarrow F' = 4 \) component of the cesium D1 line (Fig. 2). In the first laser pulse, atoms are projected on a nonabsorbing “dark” coherent superposition of the five ground-state Zeeman sublevels with magnetic quantum numbers \( m_F = -4, -2, 0, 2, \) and 4 of the \( 6S_{1/2}(F=4) \) ground state. This coherent superposition is probed with a second projection pulse after a time \( T \), at which interference is observed.

Let us derive the interference pattern for a more general experiment with \( N \) paths, which can be realized using a transition from a ground state with total angular momentum \( F = N - 1 \) to an excited state with total angular momentum \( F' = F \). A single dark state with \( N \) components exists, and the atom in the first optical pulse is projected on this coherent superposition of ground states with only even (or only odd) magnetic quantum numbers. At a time \( t \) after the pulse, the dark state has evolved into \( |\varphi_d(t)\rangle = \sum_{m_F} c_{m_F} e^{-i\delta} |g_{m_F}\rangle \), where \( |g_{m_F}\rangle \) denotes a ground state of magnetic quantum number \( m_F \). The weights \( c_{m_F} \) are such that \( |\varphi_d(t)\rangle \) is dark at \( t = 0 \). Let us assume equal field amplitudes for the \( \sigma^+ \) and \( \sigma^- \) polarized laser fields at all times, which will result in a symmetric dark state. The Raman-like two-photon detuning is \( \delta = \omega_+ - \omega_- - \omega_A \), where \( \omega_{\pm} \) denotes the frequencies of the \( \sigma_\pm \) polarized wave (\( \omega_+ = \omega_- \)) and \( \omega_A \) the Zeeman splitting between two adjacent even (or odd) \( m_F \) levels [15]. The phase shift due to the tensor polarizing (TP) pulsed optical field gives rise to the additional magnetic sublevel-dependent phase shift of Eq. (1). When using \( \pi \)-polarized light, the matrix elements for an \( F \rightarrow F' \)

\[
\Omega_{m_F}^2 = \Omega_{m_F}^2 / 4 \Delta \text{ in the far-detuned limit with } \Delta \text{ as the laser detuning and } \Omega_{m_F} \text{ as the corresponding Rabi frequency. We derive a nonlinear phase shift } \Delta \varphi_{m_F} = m_F^2 \Omega_{m_F}^{-1} \tau / 4 \Delta, \text{ which one can relate to the intensity of the tensor polarizing beam } I_{TP} \text{ after some algebra, using}
\]

\[
\Omega_{m_F=1}^2 = I_{TP} \{ 8 \pi e^2 \Gamma / h \omega^3 [1 / (1 + 2S)^2] \}
\]

with the natural linewidth \( \Gamma \) and the nuclear spin \( S \).

At time \( t = T \) the second Ramsey pulse is applied with the phase of one of its beams shifted by \( \theta \). With \( \theta = 0 \) and for no additional phase shifts, the atom by the time of the second pulse is still dark for the light field. In general, the atom by this time, however, is in a superposition of the dark and the coupled states. The second Ramsey pulse will remove most of the population that is not dark and optically pump these atoms into another hyperfine level (e.g., the \( F = 3 \) ground state for the scheme of Fig. 2), which is not detected anymore. The part of the wave function that remains in the dark state is given by the projection

\[
\langle \varphi_d(T) | \varphi_d(0) \rangle = \sum_{n=0}^{F} c_n^2 \exp \left[ -i \left( n(T\delta + \theta) + (2n - F)^2 \frac{\Omega_{m_F=1}^2}{4 \Delta} \tau \right) \right],
\]

where we have neglected both the fraction of atoms that are repumped into the dark state by the second pulse and phase shifts due to other hyperfine sublevels. The calculated fringe pattern for our experiment with five paths is shown in Fig. 3 as a function of the intensity of the TP light. A rich structure of collapse and revival effects is expected with applied TP light.
FIG. 3. Calculated interference signal for an $F = 4$ to $F' = 4$ transition as function of both the phase of the second Ramsey pulse $\theta$ and the tensor polarizing beam intensity $I_{TP}$. The vertical scale gives the ratio $|\langle \varphi_f(T) | \varphi_d(0) \rangle|^2$ of atoms in the dark state after the pulses to those initially in that state.

The experimental setup is similar to that used in our previous work [8,10]. A thermal cesium atomic beam enters a magnetically shielded optical interaction region with a homogeneous 10-mG magnetic bias field oriented along the optical Ramsey beams. These beams are generated from a Ti:sapphire laser and pass several acousto-optical modulators (AOMs) before being spatially overlapped and expanded. The phase of the drive frequency of one AOM can be varied during the pulse sequence to allow a change of the phase of the corresponding optical beam in the second Ramsey pulse. The linearly polarized light for the optical TP beam is derived from the same laser and directed oppositely to the atomic beam. Its beam diameter of 15 mm is a factor of 5 above the largest width (vertical) of the atomic beam. The number of atoms left in the dark state after the second optical pumping pulse is measured by irradiating the atoms with an optical beam tuned to the $6S_{1/2}(F = 4) - 6P_{3/2}(F' = 5)$ cycling transition and collecting the resulting fluorescence. This beam is generated from a diode laser and is copropagating with the thermal atomic beam to allow Doppler selection of slowly moving atoms. For the typically detected atomic velocity class (around 200 m/s), the TP light appears about 400 MHz blue detuned from the $6S_{1/2}(F = 4) - 6P_{3/2}(F' = 4)$ transition.

Typical interference signals with the two Ramsey pulses separated by $T = 63 \mu s$ and pulse lengths of 1.3 $\mu s$ are shown in Fig. 4 for different intensities of the TP beam. The interaction with the TP light takes place in between the Ramsey pulses for a period $\tau = 60 \mu s$. For $I_{TP} = 0$, one observes an Airy-function-like interference signal with good contrast and a fringe width of 0.20×2 $\pi$, which is reasonably close to the theoretical width of 0.16×2 $\pi$. With applied TP beam a phase shift quadratic in the path number is introduced and the phase difference between neighboring paths is no longer constant, causing collapse, period doubling, partial revival effects, and total revival for different values of the laser intensity. The (first) complete revival occurs when the phase difference between $m_F = 0$ and $m_F = \pm 2$ equals $2 \pi$, and $8 \pi$ between $m_F = 0$ and $m_F = \pm 4$, causing all magnetic sublevels to interfere constructively with zero phase of the second Ramsey pulse. This revival is observed near an intensity $I_{TP} = 43$ mW/cm², which, accounting for a 20% absolute calibration uncertainty, compares with the theoretically estimated value $I_{revival} = 33$ mW/cm² (including all hyperfine levels). A signal shifted in phase by $\pi$ is measured at $I_{TP} = 19$ mW/cm². One expects this shifted pattern to occur at half the intensity for the total revival, yielding an induced phase difference of $\pi$ between $m_F = 0$ and $m_F = \pm 2$, and $4 \pi$ between $m_F = 0$ and $m_F = \pm 4$. Constructive interference of all sublevels is only obtained when applying the final pulse with a $\pi$ phase shift. Figure 3 shows that besides full revivals partial revivals with roughly 70% amplitude of the full signal size are also expected. A partial revival is observed near $I = 11$ mW/cm². The experimental fringe patterns for larger intensities of the TP beam generally have a reduced contrast. We attribute this mainly to spatial variations in the intensity of the Gaussian TP beam over the vertical profile of the atomic beam, leading to a residual dependence of the tensorial phase shift on the atomic trajectory. On the other hand, intensity gradients along the atomic velocity are comparatively small, as the optical TP beam is directed longitudinally to the atomic beam, such that phase shifts due to classical forces are expected to be suppressed by at least two orders of magnitude. To a smaller extent, the fringe contrast is also reduced by the finite width of the selected velocity slice for signal detection (about 18 MHz Doppler width), since the ac Stark shift experienced by the atoms depends on the Doppler shift of the TP light.

In a brief view of the experiment, one could argue that during the process of switching on and off the laser, electric-field transient intensity gradients occur, which do result in classical forces. One can, however, show that the resulting phase shift is smaller than that given in Eq. (1) by a factor $\nu/v$, where $\nu$ denotes the atomic velocity [6]. For experiments with thermal atoms, the phase shift from the switching
process is thus completely negligible. We have experimentally verified that the position of the revivals indeed depends on the product of the TP-laser intensity $I_{TP}$ and pulse time $t$, and not on $I_{TP}$ alone, which one expects if the quadratic phase shift is due to field gradients during the switching process.

We have performed similar experiments on the $6S_{1/2}(F = 3) - 6P_{1/2}(F' = 3)$ transition, where the dark state is composed of only four Zeeman sublevels with $m_F = -3, -1, 1,$ and 3. The results are shown in Fig. 5. The interference signal at the complete revival (at $I_{TP} = 13.8$ mW/cm$^2$) has a much higher contrast than that observed for the transition $F = 4$ to $F' = 4$. We attribute this to the fewer Zeeman levels involved here, which implies that at the total revival the phase difference between the central and outermost Zeeman sublevels equals only $2\pi$. Any intensity imperfections of the TP beam are thus expected to be roughly a factor of 4 less severe. However, the smaller number of contributing $m_F$ levels also results in a less rich topology.

From a different perspective, the described experiments can also be interpreted as a quantum-nondemolition measurement of the number of photons in the TP-light field [17]. From the measured interference pattern the quadratic phase shift can be derived, giving a measure for the intensity in the $\pi$-polarized mode without altering the number of photons in the light field.

To conclude, we have measured collapse and revival effects due to a nonlinear phase shift induced by the tensor contribution of the atomic polarizability using multiple beam Ramsey interference. This can be interpreted as an internal state version of an Aharonov-Bohm experiment. Multiple beam Ramsey spectroscopy holds promise for future experiments testing for a permanent electric dipole of an atom [electric dipole moment (EDM)] while applying a strong static electric field. Besides the higher resolution compared to experiments measuring the difference between two adjacent Zeeman sublevels, this technique also has advantages in terms of systematic effects. The quadratic Stark effect as a major source of potential systematic uncertainties in most atomic EDM experiments is expected to cause less systematic errors in our scheme, since terms linear and nonlinear in $m_F$ can be measured separately. This point will become increasingly important for future EDM experiments performed in optical dipole traps, where the trapping beam induces an additional tensorial ac Stark shift [18].

[15] A finite splitting $\omega_\Delta$ is of importance when using elliptically polarized tensor polarizing light to inhibit Raman transitions between the $m_F$ levels.